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Self-consistent charge dynamics in magnetized dusty plasmas: Low-frequency electrostatic modes

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Charges on the dust grains immersed in a plasma vary in time. We considered such a variation in the case of a magnetoplasma. We follow the hydrodynamic approach to obtain dispersion relations for some low-frequency electrostatic modes that exist in magnetized dusty plasmas, taking into account charge fluctuation on the dust. Using self-consistent charge dynamics we obtain dielectric permittivity tensor for magnetized dusty plasmas. We use this permittivity to investigate the low-frequency electrostatic modes in magnetized dusty plasmas. Our analysis shows that the presence of the dust grains in magneto-plasma causes different effects in different frequency regimes.

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I. INTRODUCTION

In recent years [1–27] dusty plasmas have attracted the attention of a large number of workers because of its applications in space plasmas (viz. ionosphere, planetary rings, cometary tails, asteroid zones) as well as in laboratory plasmas [viz. plasmas in magnetohydrodynamic (MHD) power generators, plasma processing, and rocket exhaust]. In contrast to ordinary plasmas, dusty plasmas contain an additional species of large-sized dust grains of radii in the range of $10^{-2}-10^{-6}$ cm. In most situations, the dust grains are found to become negatively charged and the magnitude of this charge is of the order of $10^3\,e$ to $10^4\,e$ for 1-μm-sized particles. The presence of these massive and highly charged particles can significantly change the collective behavior of the plasma in which they are suspended. Consequently, a great deal of work on this aspect of dusty plasmas has been carried out in the past [1–10]. Most of the existing theoretical analyses of dusty plasmas assume the charge on the dust particle to be constant. Earlier some workers had pointed out variation in the dust charge [1–4]. However, study of plasma waves and instabilities incorporating dust charge variation has begun only recently [11–15]. Jana, Sen, and Kaw [14] have treated the charge on the dust particles as a time-dependent dynamical variable, and have shown that this fluctuation leads to a different dissipative mechanism of the dust-acoustic mode and instabilities of streaming ion mode. Bhatt and Pandey [15] pointed out an inconsistency in an earlier analysis, and added sink terms in electron and ion number conservation equations. Since then the grain charge fluctuation has also been verified experimentally [16–18]. Computer simulations have also been conducted to verify the above [19,20].

While many studies have been conducted for unmagnetized dusty plasmas, only a few authors have studied dusty plasmas in the presence of a magnetic field [21–23]. However, none of them have incorporated the time variation in the dust charge. Since the time-dependent currents which are associated with the self-consistent electric and magnetic fields of plasma modes flow onto the surface of the dust particle, the dust charge in the presence of plasma modes becomes a time-dependent variable. In this paper, we have theoretically investigated the low-frequency electrostatic modes in magnetized dusty plasma, taking into account this time dependence of the dust charge.

Our theory is based on the validity of the following assumptions: (i) The electron and ion plasma between the grains can be described by a Maxwellian distribution of the velocity. (ii) The dust grain charging is due to electron and ion collection currents only, and other charging processes, viz. photoemissions, secondary emissions and field emissions are negligible. (iii) The external static magnetic field B_0 is weak so that $a/r_{Be}<1$ (where a is the grain radius and r_{Be} is the gyroradius of electrons). This point will be further elaborated in Sec. IV. (iv) The dust particles are spherical in shape and have the same radii. This is a reasonable assumption provided that the size of the grains lies in a narrow range. (v) Dust grains are not very densely packed, i.e., the number density of electrons and ions is much greater than that of the dust. (vi) The frequency of the waves is low, i.e., $\lambda \gg \lambda_D \gg a$, where λ is the wavelength of the wave and λ_D is the Debye length of the ambient plasma. In this range of frequencies, effects due to different shapes, sizes, and materials of grains may be ignored.

Using the above-mentioned assumptions and a general formalism which involves solving hydrodynamic equations and Maxwell's equations, we obtain a general expression for the dispersion relation for electrostatic waves in magnetized dusty plasmas. We use the general expression of the dispersion relation to study three low-frequency electrostatic modes, viz. dust-cyclotron, ion-cyclotron, and lower-hybrid modes. Our analysis shows that the presence of the charged dust grains leads to the appearance of additional and/or modification of existing plasma modes in the low-frequency regime arising from dust grain dynamics. Dust charge fluctuation leads to the mechanism of collisionless damping in all low-frequency modes discussed in this paper.

Physically, the fluctuation of charge on the dust grains modifies the total current density in the plasma, perturbation in which now has two components. One of them is 90° out of phase with the electric field of the wave and is the usual term, while the other, which is in phase with the electric field and arises only due to the effect of charge fluctuation, leads to the damping of the wave.

II. BASIC EQUATIONS

Let us consider a dusty plasma to be a three-component plasma consisting of electrons, singly charged ions, and spherical particles of uniform radii. Under suitable conditions described in Sec. I, electron and ion currents to the dust grains immersed in magnetoplasma [25-28,14] are given by

$$I_e = -\pi a^2 e \left[\frac{8kT_e}{\pi m_e}\right]^{1/2} n_e \exp\left[\frac{e(\phi_f - V)}{kT_e}\right] , \qquad (1)$$

$$I_i = \pi a^2 e \left[\frac{8kT_i}{\pi m_i}\right]^{1/2} n_i \left[1 - \frac{e(\phi_f - V)}{kT_i}\right] . \qquad (2)$$

Here a is the radius of the grain, and T_e (T_i), m_e (m_i), and n_e (n_i) are temperature, mass, and number density of electrons (ions). $\phi_f - V$ is the difference between the grain potential and the plasma potential. The above equations have been written for dust grains which become negatively charged due to larger electron flux than ion flux, and have a much smaller thermal velocity than electron/ion thermal velocities.

In equilibrium, $I_{e0} + I_{i0} = 0$. The dust charge fluctuation is governed by

$$\frac{d}{dt} q_{d1} = I_{e1} + I_{i1} , \qquad (3)$$

where I_{e1} and I_{i1} are perturbed electron and ion currents into the dust grain given by (subscript 1 throughout refers to first order perturbation over corresponding equilibrium value)

$$I_{e1} = -|I_{e0}| \left[\frac{n_{e1}}{n_{e0}} + \frac{e\phi_{f1}}{T_e}\right] , \qquad (4)$$

$$I_{i1} = |I_{i0}| \left[\frac{n_{i1}}{n_{i0}} - \frac{e\phi_{f1}}{w_0}\right] . \qquad (5)$$

Here ϕ_{f1} is the fluctuation in the dust grain potential given by $\phi_{f1} = q_{d1}/C$, where C is the capacitance of the dust grain and $w_0 = T_i - e\phi_{f0}$, ϕ_{f0} being the equilibrium floating potential.

Substituting Eqs. (4) and (5) into (3), we obtain

$$\frac{d}{dt} q_{d1} + \eta q_{d1} = |I_{e0}| \left[\frac{n_{i1}}{n_{i0}} - \frac{n_{e1}}{n_{e0}}\right] , \qquad (6)$$

where

$$\eta = \frac{e|I_{e0}|}{C} \left[\frac{1}{T_e} + \frac{1}{w_0}\right] .$$

Propagation of waves in cold magnetized dusty plasmas is governed by Maxwell's equations (assuming perturbation to vary as $e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}$)

$$\nabla \times \mathbf{E}_1 = \frac{i\omega}{c} \mathbf{B}_1 , \qquad (7)$$

$$\nabla \times \mathbf{B}_1 = -\frac{i\omega}{c} \mathbf{E}_1 + \frac{4\pi}{c} \mathbf{J}_1 , \qquad (8)$$

$$\mathbf{J} = \sum_\alpha q_\alpha n_\alpha \mathbf{V}_\alpha , \quad \alpha = e , i , \text{ or } d . \qquad (9)$$

The linearized number density conservation equations for the electron, ion, and dust are (assuming the drift velocity to be zero, i.e., $\mathbf{V}_{\alpha 0} = \mathbf{0}$)

$$\frac{\partial}{\partial t} n_{a1} + \nabla\cdot(n_{a0} \mathbf{V}_{a1}) = -\beta_a n_{a1} , \quad a = e \text{ or } i , \qquad (10)$$

$$\frac{\partial}{\partial t} n_{d1} + \nabla\cdot(n_{d0} \mathbf{V}_{d1}) = 0 . \qquad (11)$$

The total charge conservation equation is

$$\nabla\cdot\mathbf{J} + \frac{\partial}{\partial t}\rho = 0 , \qquad (12)$$

where

$$\beta = \frac{|I_{e0}|}{e} \frac{n_{d0}}{n_{e0}} , \quad \beta_e = \beta , \quad \beta_i = \beta\delta , \quad \delta = \frac{n_{e0}}{n_{i0}} , \qquad (13)$$

The equations of motion of the electrons, ions, and dust are

$$m_e \frac{\partial}{\partial t} \mathbf{V}_{e1} = -e\mathbf{E}_1 - \frac{e}{c} \mathbf{V}_{e1} \times \mathbf{B}_0 - m_e \beta \mathbf{V}_{e1} , \qquad (14)$$

$$m_i \frac{\partial}{\partial t} \mathbf{V}_{i1} = e\mathbf{E}_1 + \frac{e}{c} \mathbf{V}_{i1} \times \mathbf{B}_0 - m_i \beta\delta \mathbf{V}_{i1} , \qquad (15)$$

$$m_d \frac{\partial}{\partial t} \mathbf{V}_{d1} = -q_{d0}\mathbf{E}_1 - \frac{q_{d0}}{c} \mathbf{V}_{d1} \times \mathbf{B}_0 . \qquad (16)$$

In Eq. (16), i.e., the equation of motion of the dust particle, we have neglected the momentum gain term as it is insignificant because of the heavy mass of the dust particle. Here $-q_{d0}$ is the equilibrium charge on the dust grain (q_{d0}+ve). Solving Eqs. (14), (15), and (16), we obtain the following expression for perturbation in the velocity:

$$\mathbf{V}_{\alpha 1} = -i(\omega + i\beta_\alpha) \frac{q_{\alpha 0}}{m_\alpha} \begin{bmatrix} \dfrac{1}{\omega_{c\alpha}^2 - (\omega + i\beta_\alpha)^2} & \pm \dfrac{i\omega_{c\alpha}}{(\omega + i\beta_\alpha)} \dfrac{1}{\omega_{c\alpha}^2 - (\omega + i\beta_\alpha)^2} & 0 \\ \mp \dfrac{i\omega_{c\alpha}}{(\omega + i\beta_\alpha)} \dfrac{1}{\omega_{c\alpha}^2 - (\omega + i\beta_\alpha)^2} & \dfrac{1}{\omega_{c\alpha}^2 - (\omega + i\beta_\alpha)^2} & 0 \\ 0 & 0 & -\dfrac{1}{(\omega + i\beta_\alpha)^2} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} . \qquad (17)$$

Here $\omega_{c\alpha}$ and $\omega_{p\alpha}$ have their usual meaning. ω_{cd} and ω_{pd} correspond to the equilibrium value of the dust charge. The upper sign is for negatively charged particles, i.e., electrons, and dust, and the lower one is for positive ions. Using continuity equations (10)–(12), (17), and Eq. (8), we obtain the general dispersion relation for electrostatic waves in magnetized dusty plasmas:

$$\mathbf{k}\cdot\overleftrightarrow{\epsilon}\cdot\mathbf{k}=0 , \tag{18}$$

where $\overleftrightarrow{\epsilon}$ is dielectric permittivity tensor for the magnetized dusty plasma, given by

$$\overleftrightarrow{\epsilon}=\overleftrightarrow{\mathbf{I}}+\overleftrightarrow{\chi}_e\left[1+\frac{i\beta}{\omega+i\eta}\right]+\overleftrightarrow{\chi}_i\left[1+\frac{i\beta\delta}{\omega+i\eta}\right]+\overleftrightarrow{\chi}_d . \tag{19}$$

Here $\overleftrightarrow{\chi}_\alpha$ are given by

$$\overleftrightarrow{\chi}_a=\begin{bmatrix} \dfrac{\omega_{pa}^2}{\omega_{ca}^2-(\omega+i\beta_a)^2} & \pm\dfrac{i\omega_{ca}}{(\omega+i\beta_a)}\dfrac{\omega_{pa}^2}{\omega_{ca}^2-(\omega+i\beta_a)^2} & 0 \\ \mp\dfrac{i\omega_{ca}}{(\omega+i\beta_a)}\dfrac{\omega_{pa}^2}{\omega_{ca}^2-(\omega+i\beta_a)^2} & \dfrac{\omega_{pa}^2}{\omega_{ca}^2-(\omega+i\beta_a)^2} & 0 \\ 0 & 0 & -\dfrac{\omega_{pa}^2}{(\omega+i\beta_a)^2} \end{bmatrix} , \tag{20}$$

$$\overleftrightarrow{\chi}_d=\begin{bmatrix} \dfrac{\omega_{pd}^2}{\omega_{cd}^2-\omega^2} & \dfrac{i\omega_{cd}}{\omega}\dfrac{\omega_{pd}^2}{\omega_{cd}^2-\omega^2} & 0 \\ -\dfrac{i\omega_{cd}}{\omega}\dfrac{\omega_{pd}^2}{\omega_{cd}^2-\omega^2} & \dfrac{\omega_{pd}^2}{\omega_{cd}^2-\omega^2} & 0 \\ 0 & 0 & -\dfrac{\omega_{pd}^2}{\omega^2} \end{bmatrix} . \tag{21}$$

Two limiting cases are of considerable interest. In one, if $B\rightarrow 0$, $\omega_c\rightarrow 0$. Then we recover Eq. (10) of Ref. [14], with

$$\chi_\alpha=-\frac{\omega_{p\alpha}^2}{(\omega+i\beta_\alpha)^2} , \quad \alpha=e,i,d , \quad \beta_d=0 , \tag{22}$$

i.e., in the absence of magnetic field our permittivity tensor reduces to the expression of the scalar permittivity as obtained by the authors of Refs. [14,15]. In the second case, if $n_{d0}\rightarrow 0$, $\beta\rightarrow 0$, we recover the usual permittivity tensor

$$\overleftrightarrow{\epsilon}=\overleftrightarrow{\mathbf{I}}+\overleftrightarrow{\chi}_e+\overleftrightarrow{\chi}_i \tag{23}$$

for the two-component plasma. Thus in the absence of dust particles the dielectric tensor for a two-component plasma consisting of electrons and ions is recovered.

The permittivity tensor can be written as

$$\overleftrightarrow{\epsilon}=\begin{bmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} , \tag{24}$$

where

$$\begin{aligned}\epsilon_1=1&-\left[\frac{\omega_{pe}^2}{(\omega+i\beta)^2-\omega_{ce}^2}\right]\left[1+\frac{i\beta}{\omega+i\eta}\right]\\&-\left[\frac{\omega_{pi}^2}{(\omega+i\beta\delta)^2-\omega_{ci}^2}\right]\left[1+\frac{i\beta\delta}{\omega+i\eta}\right]\\&-\left[\frac{\omega_{pd}^2}{\omega^2-\omega_{cd}^2}\right] ,\end{aligned} \tag{25}$$

$$\begin{aligned}\epsilon_3=1&-\left[\frac{\omega_{pe}^2}{(\omega+i\beta)^2}\right]\left[1+\frac{i\beta}{\omega+i\eta}\right]\\&-\left[\frac{\omega_{pi}^2}{(\omega+i\beta\delta)^2}\right]\left[1+\frac{i\beta\delta}{\omega+i\eta}\right]-\left[\frac{\omega_{pd}^2}{\omega^2}\right] ,\end{aligned} \tag{26}$$

etc.

In Sec. III, we will use Eq. (18) to study some low-frequency electrostatic modes in magnetized dusty plasmas.

III. DISPERSION RELATION

The dispersion relation for electrostatic waves propagating in the y-z plane is

$$\epsilon_1\left[\frac{k_\perp}{k}\right]^2+\epsilon_3\left[\frac{k_\parallel}{k}\right]^2=0\ , \tag{27}$$

where ε_1 and ε_3 are given by Eqs. (25) and (26). We will study this dispersion relation for different frequency regimes.

Case I. First, let us investigate the lowest-frequency regime, i.e., when $\omega\approx\omega_{cd}$, so that $\omega\ll\omega_{ci}, \omega_{ce}$. We then obtain the dispersion relation

$$\omega=\omega_r+i\omega_i\ , \tag{28}$$

where ω_r and ω_i are given by

$$\left[\frac{\omega_r}{\omega_{cd}}\right]^2=\frac{1}{2}\left\{1+\frac{\dfrac{\omega_{pd}^2}{\omega_{cd}^2}\sin^2\theta+\dfrac{\omega_{pe}^2+\omega_{pi}^2}{\omega_{cd}^2}\cos^2\theta}{1+\left[\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}\right]\sin^2\theta}\pm\left[\left[1+\frac{\dfrac{\omega_{pd}^2}{\omega_{cd}^2}\sin^2\theta+\dfrac{\omega_{pe}^2+\omega_{pi}^2}{\omega_{cd}^2}\cos^2\theta}{1+\left[\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}\right]\sin^2\theta}\right]^2 -4\left[\frac{\dfrac{\omega_{pe}^2+\omega_{pi}^2}{\omega_{cd}^2}}{1+\left[\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}\right]\sin^2\theta}\right]\cos^2\theta\right]^{1/2}\right\}, \tag{29}$$

$$\omega_i=-\frac{\beta}{2}\left[\frac{\left[\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}\delta\right]\sin^2\theta+\left[\dfrac{\omega_{pe}^2+\omega_{pi}^2\delta}{\omega_r^2}\right]\cos^2\theta}{\left[\dfrac{\omega_r^2\omega_{pd}^2}{(\omega_r^2-\omega_{cd}^2)^2}\right]\sin^2\theta+\left[\dfrac{\omega_{pe}^2+\omega_{pi}^2}{\omega_r^2}\right]\cos^2\theta}\right]. \tag{30}$$

For near normal propagation, $k_\perp/k\approx1, k_\parallel/k=\cos\theta$ (small), we obtain

$$\left[\frac{\omega_r}{\omega_{cd}}\right]^2=\frac{1}{2}\left\{1+\frac{\dfrac{\omega_{pd}^2}{\omega_{cd}^2}+\left[\dfrac{\omega_{pe}^2+\omega_{pi}^2}{\omega_{cd}^2}\right]\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}}\pm\left[\left[1+\frac{\dfrac{\omega_{pd}^2}{\omega_{cd}^2}+\dfrac{\omega_{pe}^2+\omega_{pi}^2}{\omega_{cd}^2}\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}}\right]^2 -4\left[\frac{\dfrac{\omega_{pe}^2+\omega_{pi}^2}{\omega_{cd}^2}}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}}\right]\cos^2\theta\right]^{1/2}\right\} \tag{31a}$$

This equation may be approximated as

$$\omega_r^2\simeq\omega_{cd}^2+\frac{\omega_{pd}^2+(\omega_{pe}^2+\omega_{pi}^2)\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}}\ , \tag{31b}$$

$$\omega_i=-\frac{\beta}{2}\left[\frac{\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}\delta+\left[\dfrac{\omega_{pe}^2+\omega_{pi}^2\delta}{\omega_r^2}\right]\cos^2\theta}{\dfrac{\omega_r^2\omega_{pd}^2}{(\omega_r^2-\omega_{cd}^2)^2}+\left[\dfrac{\omega_{pe}^2+\omega_{pi}^2}{\omega_r^2}\right]\cos^2\theta}\right]. \tag{32}$$

To understand the nature of dispersion relation, let us consider the case when $\theta=90°$,

$$\omega=\pm\left[\omega_{cd}^2+\frac{\omega_{pd}^2}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}}\right]^{1/2}-\frac{i\beta}{2}\,\frac{\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}\delta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}}\ . \tag{33}$$

Equations (31b) and (33) suggest that dust particles are providing the inertia in the fluid of electrons and ions. This mode is called the dust-cyclotron mode. Variation of ω_i with θ is given by Eq. (32). From this equation one finds that for $\omega \approx \omega_{cd}$ damping is almost nil for perpendicularly propagating dust-cyclotron waves. For a dense plasma this happens when $\omega_{pe}\cos\theta \approx \omega_{pd}$.

Case II. We next study the frequency range $\omega \approx \omega_{ci}$, then $\omega_{cd} \ll \omega \ll \omega_{ce}$. For this case the following dispersion relation is obtained:

$$\left[\frac{\omega_r}{\omega_{ci}}\right]^2 = \frac{1}{2}\left\{1+\frac{\dfrac{\omega_{pd}^2+\omega_{pi}^2\sin^2\theta}{\omega_{ci}^2}+\dfrac{\omega_{pe}^2}{\omega_{ci}^2}\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\sin^2\theta} \pm \left[\left[1+\frac{\dfrac{\omega_{pd}^2+\omega_{pi}^2\sin^2\theta}{\omega_{ci}^2}+\dfrac{\omega_{pe}^2}{\omega_{ci}^2}\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\sin^2\theta}\right]^2 - 4\left[\frac{\dfrac{\omega_{pd}^2+\omega_{pe}^2\cos^2\theta}{\omega_{ci}^2}}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\sin^2\theta}\right]\right]^{1/2}\right\}, \tag{34}$$

$$\omega_i = -\frac{\beta}{2}\left[\frac{\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\sin^2\theta+\dfrac{\omega_{pi}^2(\omega_{ci}^2+\omega_r^2)}{(\omega_r^2-\omega_{ci}^2)^2}\delta\sin^2\theta+\dfrac{\omega_{pe}^2}{\omega_r^2}\cos^2\theta}{\dfrac{\omega_{pd}^2}{\omega_r^2}+\dfrac{\omega_{pi}^2\omega_r^2}{(\omega_r^2-\omega_{ci}^2)^2}\sin^2\theta+\dfrac{\omega_{pe}^2}{\omega_r^2}\cos^2\theta}\right]. \tag{35}$$

For near normal propagation, $k_\perp/k \approx 1$. Thus for $k_\parallel/k=\cos\theta$ (small),

$$\left[\frac{\omega_r}{\omega_{ci}}\right]^2 = \frac{1}{2}\left\{1+\frac{\dfrac{\omega_{pd}^2+\omega_{pi}^2}{\omega_{ci}^2}+\dfrac{\omega_{pe}^2}{\omega_{ci}^2}\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}} \pm \left[\left[1+\frac{\dfrac{\omega_{pd}^2+\omega_{pi}^2}{\omega_{ci}^2}+\dfrac{\omega_{pe}^2}{\omega_{ci}^2}\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}}\right]^2 - 4\left[\frac{\dfrac{\omega_{pd}^2+\omega_{pe}^2\cos^2\theta}{\omega_{ci}^2}}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}}\right]\right]^{1/2}\right\} \tag{36a}$$

$$\simeq 1+\frac{\dfrac{\omega_{pd}^2+\omega_{pi}^2}{\omega_{ci}^2}+\dfrac{\omega_{pe}^2}{\omega_{ci}^2}\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}}, \tag{36b}$$

$$\omega_i = -\frac{\beta}{2}\left[\frac{\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2(\omega_{ci}^2+\omega_r^2)}{(\omega_r^2-\omega_{ci}^2)^2}\delta+\dfrac{\omega_{pe}^2}{\omega_r^2}\cos^2\theta}{\dfrac{\omega_{pd}^2}{\omega_r^2}+\dfrac{\omega_{pi}^2\omega_r^2}{(\omega_r^2-\omega_{ci}^2)^2}+\dfrac{\omega_{pe}^2}{\omega_r^2}\cos^2\theta}\right]. \tag{37}$$

For $\theta=90°$ we obtain

$$\omega = \pm\left[\omega_{ci}^2+\frac{\omega_{pi}^2}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}}\right]^{1/2} - \frac{i\beta\delta}{2}\left[\frac{2+2\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}+\dfrac{\omega_{pi}^2}{\omega_{ci}^2}}\right]. \tag{38}$$

Equations (36b) and (37) describe the ion-cyclotron mode in the presence of charged dust. We note that ω_r is slightly increased by the presence of the dust grain. The effect of charge fluctuation is described by Eq. (37). When $\omega \approx \omega_{ci}$, $\omega_i \approx -\beta\delta$, which depends on dust and ion number densities.

Case III. Finally we consider frequencies such that ω_{cd} and $\omega_{ci} \ll \omega \ll \omega_{ce}$. For this case we obtain the following dispersion relation:

$$\omega_r = \left[\frac{\omega_{pd}^2+\omega_{pi}^2+\omega_{pe}^2\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\sin^2\theta}\right]^{1/2}, \tag{39}$$

$$\omega_i = -\frac{\beta}{2}\left[\frac{\omega_{pi}^2\delta+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\omega_r^2\sin^2\theta+\omega_{pe}^2\cos^2\theta}{\omega_{pd}^2+\omega_{pi}^2+\omega_{pe}^2\cos^2\theta}\right]. \tag{40}$$

For near normal propagation, $k_\parallel/k \ll 1, k_\perp/k \approx 1$. Thus for $k_\parallel/k=\cos\theta$ (small),

$$\omega_r=\left[\frac{\omega_{pd}^2+\omega_{pi}^2+\omega_{pe}^2\cos^2\theta}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}}\right]^{1/2} . \tag{41a}$$

This can be rewritten as

$$\omega_r^2=\omega_{\rm LH}^2\left[1+\frac{\omega_{pd}^2}{\omega_{pi}^2}+\frac{\omega_{pe}^2}{\omega_{pi}^2}\cos^2\theta\right] , \tag{41b}$$

where

$$\omega_{\rm LH}^2=\frac{\omega_{pi}^2}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}} ,$$

$$\omega_i=-\frac{\beta}{2}\left[\frac{\omega_{pi}^2\delta+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\omega_r^2+\omega_{pe}^2\cos^2\theta}{\omega_{pd}^2+\omega_{pi}^2+\omega_{pe}^2\cos^2\theta}\right] . \tag{42}$$

From Eqs. (39) and (40) we can see that when $\cos\theta$ is not very small $[\gg(m_e/m_i)]$, ω_{pi}^2 and ω_{pd}^2 may be ignored, and electrons oscillate around the magnetic field with frequency $\omega_{pe}\cos\theta$, the oscillations decaying with time $[\omega_i=-(\beta/2)]$. When small $k_\parallel$ is present, we obtain lower-hybrid waves in the presence of dust [Eqs. (41a), (41b), and (42)].

For $\theta=90°$, we obtain (ignoring ω_{pd} as compared to ω_{pi})

$$\omega=\frac{\omega_{pi}}{\left[1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\right]^{1/2}}-\frac{i\beta}{2}\left[\frac{\delta+(\delta+1)\dfrac{\omega_{pe}^2}{\omega_{ce}^2}}{1+\dfrac{\omega_{pe}^2}{\omega_{ce}^2}}\right] . \tag{43}$$

When $k_\parallel=0$, we have lower-hybrid resonance, as described by Eq. (43) ($\omega_{pd}\ll\omega_{pi}$),

$$\omega_r=\omega_{\rm LH}\simeq\omega_{pi} ,\quad \omega_i=-\frac{\beta\delta}{2}\quad\left[\text{if } \delta\gg\frac{\omega_{pe}}{\omega_{ce}}\right] .$$

For a dense plasma, i.e., when $(\omega_{pe}/\omega_{ce})\gg1$,

$$\omega_r=\omega_{\rm LH}\simeq\sqrt{\omega_{ce}\omega_{ci}} ,\quad \omega_i\simeq-\frac{\beta}{2}(1+\delta) .$$

IV. DISCUSSION

We have derived a general expression for the dielectric permittivity tensor for a three-component plasma consisting of electrons, ions, and dust particles in the presence of a homogeneous static magnetic field incorporating a self-consistent grain-charge fluctuation. Following a general formalism which involves solving hydrodynamic equations and Maxwell's equations, we obtained explicit expressions for the dispersion relation for three low-frequency electrostatic modes: dust-cyclotron, ion-cyclotron, and lower-hybrid modes. Looking at our expressions for the dispersion relation for various modes, i.e., Eqs. (28)–(43), the following general observations can be made.

(i) All the modes studied are damped, and the damping in all these modes is caused only on account of grain-charge fluctuation. The rate of damping is almost directly proportional to the dust number density. In the absence of charge fluctuation, damping disappears.

(ii) Taking into account the charge fluctuation does not have any effect on the real part of the frequency (ω_r).

(iii) The frequency increases with increasing dust concentration for all three modes.

(iv) The effect of dust grain dynamics becomes more significant when $k_\parallel/k$ is very small, i.e., $\theta\simeq\pi/2$.

However, the presence of highly charged massive dust particles also causes different effects in different frequency regimes. Whereas the presence of dust particles leads to the birth of the dust-cyclotron mode in the lowest-frequency regime, in the ion-cyclotron and lower-hybrid range of frequencies the presence of dust causes an increase in the frequency of existing modes.

It is easy to see from Eq. (19) itself that if we ignore fluctuation in the dust charge, the dielectric permittivity tensor reduces to that for a usual multispecies plasma. Also, in the absence of magnetic field this tensor reduces to the expressions of scalar permittivity as obtained by Jana, Sen, and Kaw [14] and Bhatt and Pandey [15].

Our theory is based on the assumption that Eqs. (1) and (2) are valid even in the presence of magnetic field. It has been shown by Chang and Spariosu [27] and Rubinstein and Laframboise [28] that dust particle charging characteristics are not significantly influenced by the existence of external magnetic field when $a/r_{Be}<1$. Thus application of our theory is restricted to the situation for which $a/r_{Be}<1$. It is not difficult to see that this condition can easily be satisfied for space plasmas (viz. planetary magnetospheres) and laboratory plasmas which are not very hot, containing 1-μm-sized dust grains. The authors of Refs. [27,28] have also discussed the situation when $a/r_{Be}>1$, and concluded that the presence of magnetic field in some cases will result only in reducing the magnitude of the equilibrium charging current by a small factor (the form of the charging expression remains the same). Thus our analysis will still hold in such cases, except the fact that the damping (growth) rate will be reduced by the same factor.

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